

Indian Statistical Institute
B.Math.(Hons.) II Year
First Semester Exam, 2006-07
Algebra III

Time: 3 hrs

Date:27-11-06

Total Marks : 50
Instructor: J Biswas

Attempt all questions.

1. Let $f(x) \in \mathbb{Z}[x]$ be a monic polynomial and let $\alpha \in \mathbb{R}$ be a rational root of $f(x)$. Prove that α must be an integer. [7]
2. Factor $1 - 3i$ into Gaussian primes. [7]
3. Determine all integer solutions to the system of equations $AX = 0$ where $A = \begin{pmatrix} 4 & 7 & 2 \\ 2 & 4 & 6 \end{pmatrix}$. [7]
4. Let V be the the abelian group generated by x, y, z with the following complete set of relations

$$\begin{aligned} 7x + 5y + 2z &= 0 \\ 3x + 3y &= 0 \\ 13x + 11y + 2z &= 0 \end{aligned}$$

Find an isomorphic direct product of cyclic groups. [7]

5. Determine the Jordan form over \mathbb{C} of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$. [7]
6. Let T be the linear operator whose matrix is $\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$. Is the corresponding $\mathbb{C}[t]$ -module cyclic? Justify your answer. [7]
7. Let $A = (a_1, \dots, a_n)^t$ be an integer column vector. Prove that there is a matrix $P \in GL_n(\mathbb{Z})$ such that $PA = (d, 0, \dots, 0)^t$ where d is the greatest common divisor of a_1, \dots, a_n . [8]